Algebra I: Backpaper Exam (online) **Time: 2 hours**
2020-2021

Notations: $\mathbb{Z}_n$ denotes the cyclic group of order n, A_n denotes the alternating group of degree n.

(1) Find the number of group homomorphisms from $\mathbb{Z}_m$ to $\mathbb{Z}_n$. (6)
(2) Let N be a normal subgroup of a group G, such that the quotient group G/N is an infinite cyclic group. Then prove that for each positive integer n, there exists a normal subgroup H of G of index n. (6)
(3) State and prove Cauchy's theorem for finite groups. (10)
(4) State Sylow's theorems. Find the number of elements of order 3 in a non-cyclic group of order 57. (4+6)
(5) Show that a group of order 72 is not simple. (8)
(6) Show that A_5 is simple. (10)